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Finite-temperature Casimir effect on the radius stabilization of non-commutative torus

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Abstract: The one-loop correction to the spectrum of Kaluza-Klein system for the ϕ^3 model on $\mathbb{R}^{1,d} \times (\mathbb{T}^2_\theta)^L$ is evaluated in the high temperature limit, where the $1+d$ dimensions are the ordinary flat Minkowski spacetimes and the L extra two-dimensional tori are chosen to be the non-commutative torus with noncommutativity θ. The corrections to the Kaluza-Klein mass formula are evaluated and used to compute the Casimir energy with the help of the Schwinger perturbative formula in the zeta-function regularization method. The results show that the one-loop Casimir energy is independent of the radius of torus if $L = 1$. However, when $L > 1$ the Casimir energy could give repulsive force to stabilize the extra non-commutative torus if $d - L$ is a non-negative even integral. This therefore suggests a possible stabilization mechanism of extra radius in high temperature, when the extra spaces are non commutative.

Keywords: Field Theories in Higher Dimensions, Superstrings and Heterotic Strings, Non-Commutative Geometry.

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1. Introduction

The Casimir effect originally suggested in 1948 [1, 2] is regarded as the contribution of a non-trivial geometry on the vacuum fluctuation of quantum electromagnetic fields. The corresponding change in the vacuum energy can produce a vacuum pressure to attract the two perfectly conducting parallel plates. This attractive force is experimentally confirmed by Sparnaay in 1958 and recently more precise measurement have been provided [3].

Appelquist and Chodos [4] in 1983 suggested that the vacuum fluctuation of higher-dimensional gravitational field may contribute an attractive Casimir force to push the size of the extra spaces in the Kaluza-Klein unified theory [5] to the Planck scale. Near the Planck scale, it is generally believed that the non-perturbative quantum gravity can stabilize the size of the extra spaces [5]. The superstring theory, as an only candidate of unified theory including the quantum gravity, is also a higher-dimensional theory [6]. Therefore it is reasonable to conjecture that the string theory could itself provide a mechanism to stabilize the extra compact space therein.

In recent investigations, an available scale which may be used to stabilize the extra compact is the noncommutativity θ_{ij} revealed in the string/M theories [7, 8, 9]. Initially, Connes, Douglas and Schwarz [7] had shown that the supersymmetric gauge theory on non-commutative torus is naturally related to the compactification

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of Matrix theory [9]. More recently, it is known that the dynamics of a D-brane [10] in the presence of a B-field can, in certain limits, be described by the non-commutative field theories [8].

Historically, it is a hope that, introducing a parameter to deform the geometry in the small spacetime would be possible to cure the quantum-field divergences, especially in the gravity theory [11]. The later works [12], however, proved that the non-commutative field theory exhibits the same divergence as the commutative one. But the field theory with noncommutativity could show some interesting properties and still deserves furthermore investigated.

A distinct characteristic of the non-commutative field theories, found by Minwalla, Raamsdonk and Seiberg [12], is the mixing of ultraviolet (UV) and infrared (IR) divergences reminiscent of the UV/IR connection of the string theory. In a recent paper [13] we also found that the noncommutativity does not affect one-loop effective potential of the scalar field theory. However, it can become dominant in the two-loop level and have an inclination to induce the spontaneously symmetry breaking if it is not broken in the tree level, and have an inclination to restore the symmetry breaking if it has been broken in the tree level.

In the paper [14] Nam used the Kaluza-Klein mass formula, which was evaluated by Gomis, Mehen and Wise [15], to compute the one-loop Casimir energy of an interacting scalar field in a compact non-commutative space of $\mathbb{R}^{1,d} \times \mathbb{T}^2_\theta$, where $1+d$ dimensions are the ordinary flat Minkowski space and the extra two dimensions are non-commutative torus with noncommutativity θ. It is the first literature which tries to use the noncommutativity as a minimum scale to protect the collapse of the extra spaces.

In the paper [16] we followed the method of [15] to evaluate the one-loop correction to the spectrum of Kaluza-Klein system for the ϕ^3 model on $\mathbb{R}^{1,d} \times (\mathbb{T}^2_\theta)^L$, where the extra dimensions are the L two-dimensional non-commutative tori. We used the correct Kaluza-Klein mass spectrum [16] to compute the Casimir energy. It then shows that when $L > 1$ the Casimir energy due to the noncommutativity could give repulsive force to stabilize the extra non-commutative tori in the cases of $d = 4n - 2$, with n a positive integral. This therefore suggests a possible stabilization mechanism for a scenario in superstring theory, where some of the extra dimensions are non commutative.

In this paper we will investigate the previous problem, while in the non-zero temperature. Note that the Casimir energy for the system in ordinary commutative spacetime is always found to attract the two perfectly conducting parallel plates for all temperature [1, 2]. So, it is interesting to see how the noncommutativity will affect the behaviors of finite-temperature Casimir energy.

Note also that the Casimir effect is null in the supersymmetry system, as the contribution from boson field is just canceled by that from fermion field [17]. The no-go theorem of supersymmetry breaking [17] tells us that the supersymmetry could not be radiatively broken, even if the gauge symmetry has been broken in the tree level [18]. However, some mechanisms shall be proposed to break the supersymmetry in superstring theory to describe the physical phenomena [6, 17]. Thus the remaining non-zero Casimir effect may be used to render the compact space stable. An interesting mechanism which could break the supersymmetry is the temperature effect [19], a natural consequence of different statistics for bosons and fermions. This is the scenario in the early epoch of the universe and is related to the present paper.

In section 2, we extend the works of Gomis, Mehen and Wise [15] to the non-zero temperature. We evaluate the one-loop correction to the spectrum of Kaluza-Klein system in the high temperature limit. The correction to the Kaluza-Klein mass formula has the additional term which resembles that of the winding states in the string theory [20], likes as the property found in the zero temperature system [15]. In section 3, the obtained spectrum is used to compute the Casimir energy with the help of the Schwinger perturbative formula in the zeta-function regularization method [21]. The result is used to analyze the effect of the high-temperature Casimir energy on the radius stabilization. We find that the one-loop Casimir energy is independent of the radius of torus if $L = 1$. So, in this case, there is no repulsive force and we have to consider the Casimir energy to the higher order, which is beyond the scope of this paper. However, when $L > 1$ the Casimir energy could give repulsive force to stabilize the extra non-commutative torus if $d - L$ is a non-negative even integral. This therefore suggests a possible stabilization mechanism for a high-temperature scenario in the Kaluza-Klein theory, where some of the extra dimensions are non commutative. The conclusion is presented in the last section.

2. Kaluza-Klein spectrum for ϕ^3 on $\mathbb{R}^{1,d} \times (\mathbb{T}^2_\theta)^L$: finite temperature

2.1 Model

It is known that the algebra of functions on a non-commutative spacetime can be viewed as an algebra of ordinary functions on the usual spacetime with the product deformed to non-commutative Moyal product [12]. Therefore, the scalar ϕ^3 theory in $\mathbb{R}^{1,d} \times (\mathbb{T}^2_\theta)^L$ spacetime can be described by the following action:

$$S = \int d^{1+d}x \; d^{2L}y \left(\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{3!}\phi \star \phi \star \phi \right), \tag{2.1}$$

in which the $\star$ operator is the Moyal product generally defined by [12]

$$f(x) \star g(x) = e^{+\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial y^\mu}\frac{\partial}{\partial z^\nu}} f(y)g(z)|_{y,z\to x}\,, \tag{2.2}$$

where $\theta_{\mu\nu}$ is a real, antisymmetric matrix which represents the noncommutativity of the spacetime, i.e. $[x^\mu, x^\nu] = i\theta^{\mu\nu}$. In eq. (2.1) the coordinates $x^0, x^1, \ldots, x^d$ represent the commutative four-dimensional Minkowski spacetime. The $2L$ extra dimensions are taken to be the L non-commutative 2-tori $\mathbb{T}^2_\theta$ whose non-commutative coordinates are described by

$$[y^1, y^2] = [y^3, y^4] = \cdots = [y^{2L-1}, y^{2L}] = i\theta\,. \tag{2.3}$$

When $L = 1$, this coordinate can be realized in string theory by wrapping a five-brane on a two-torus $\mathbb{T}^2$ with a constant B-field along the torus. The low energy effective four-dimensional theory resulting from compactification on a non-commutative space is local and Lorentz invariant, hence it can be relevant phenomenologically [15].

2.2 One-loop calculation

At zero temperature, the momentum in the $1 + d$ Minkowski spacetime, denoted as p, is a continuous variable. However, the momenta along the tori are quantized as $\vec{k} \to \vec{k}/R$, where $\vec{k} = (k_1, \ldots, k_{2L})$ are the integrals. Therefore, using the Feynman rule [15], which includes the propagator $i\Delta(p, \vec{n}) = \frac{i(1-\delta_{\vec{n},0})}{p^2-\vec{n}^2-m^2}$ for the field with momentum $(p, \vec{n})$, and vertex function $V(\vec{k}, \vec{n}, \vec{m}) = -i\lambda\ \cos(\frac{\theta}{2R^2}\vec{n} \wedge \vec{k})\delta_{\vec{k}+\vec{n}+\vec{m},0}$ for the three legs with incoming momenta $\vec{k}, \vec{n}$ and $\vec{m}$, respectively, the one-loop contribution to the two point functions is

$$\begin{aligned}
&\frac{\lambda^2}{4}\frac{1}{(2\pi R)^{2L}}\sum_{\vec{k}}\int \frac{d^{1+d}l}{(2\pi)^{1+d}}\frac{\cos^2(\theta\ \vec{n}\wedge\vec{k}/(2R^2))(1-\delta_{\vec{k},0})(1-\delta_{\vec{n}+\vec{k},0})}{(l^2-\vec{k}^2/R^2-m^2)((l+p)^2-(\vec{n}+\vec{k})^2/R^2-m^2)} = \\
&\quad = \frac{\lambda^2}{4}\frac{1}{(2\pi R)^{2L}}\times \\
&\quad\times \sum_{\vec{k}}\int \frac{d^{1+d}l}{(2\pi)^{1+d}}\frac{1-2\,\delta_{\vec{k},0}-2\delta_{\vec{n}+\vec{k},0}+\cos(\theta\ \vec{n}\wedge\vec{k}/R^2)}{(l^2-\vec{k}^2/R^2-m^2)((l+p)^2-(\vec{n}+\vec{k})^2/R^2-m^2)}\,.
\end{aligned} \tag{2.4}$$

in which $\vec{n} \wedge \vec{k} \equiv (n_1k_2 - n_2k_1) + (n_3k_4 - n_4k_3) + \cdots + (n_{2L-1}k_{2L} - n_{2L}k_{2L-1})$. Note that l denotes the loop momenta along the non-compact directions, while $\vec{k}$ is loop momenta along compact directions. Similarly, $p\,(\vec{n})$ is the external momenta along the non-compact (compact) direction. The derivation of eq. (2.4) has used the half angle formula for the cosine and the property that $\vec{n} \wedge \vec{n} = 0$, as detailed by Gomis, Mehen and Wise [15].

The first term, second term and third term in eq. (2.4) are all divergent [15]. They are irrelevant to our investigation and will not be discussed furthermore. The last term contains a oscillatory factor $\cos(\theta\,\vec{n} \wedge \vec{k}/R^2)$ which makes the non-planar correction term to be ultraviolet finite [15].

Let us turn to the system at non-zero temperature. It is known that the divergences at non-zero temperature are the same as those at zero temperature. Therefore, to find the one-loop correction of the Kaluza-Klein spectrum at finite temperature we only need to evaluate the last term in eq. (2.4).

At temperature T $(= 1/\beta)$ the non-planar correction to the one-loop self energy is the last term in the eq. (2.4) after substituting the continuous variable l_0 by $2\pi l_0/\beta$, where the new variable l_0 is an integral. Therefore the one-loop self energy is

$$\Sigma(p,\vec{n}) = \frac{\lambda^2}{4}\frac{1}{(2\pi R)^{2L}}\frac{1}{\beta}\sum_{\vec{k}}\sum_{l_0}\int\frac{d^d\vec{l}}{(2\pi)^d}\times \tag{2.5}$$
$$\times\frac{\cos(\theta\;\vec{n}\wedge\vec{k}/R^2)}{[(2\pi l_0/\beta)^2-\vec{l}^2-\vec{k}^2/R^2-m^2][(\frac{2\pi}{\beta}(l_0+p_0))^2-(\vec{l}+\vec{p})^2-(\vec{n}+\vec{k})^2/R^2-m^2]}\,.$$

To proceed, after introducing the Feynman parameter x we can perform the integration over the momentum $\vec{l}$, then expressing the result in terms of the Schwinger parameter α the equation becomes

$$\begin{aligned}\Sigma(p,\vec{n}) &= \frac{\lambda^2}{4}\frac{1}{(2\pi R)^{2L}}\frac{1}{\beta}\frac{1}{(4\pi)^{2/d}}\sum_{\vec{k}}\sum_{l_0}\int_0^1 dx\int_0^\infty d\alpha\alpha^{1-d/2}\times\\ &\times\exp\left\{-\alpha\left[m^2+x(1-x)\vec{p}^2+\frac{\vec{k}^2+x(2\vec{k}\cdot\vec{n}+\vec{n}^2)}{R^2}+\right.\right.\\ &\qquad\left.\left.+\frac{l_0^2+x(2l_0p_0+p_0^2)}{(1/2\pi\beta)^2}\right]\right\}\times\\ &\times\frac{1}{2}\left\{\prod_{i=1}^{2L-1}\exp\left[i\frac{\theta}{R^2}(n_ik_{i+1}-n_{i+1}k_i)\right]+\right.\\ &\qquad\left.+\prod_{i=1}^{2L-1}\exp\left[-i\frac{\theta}{R^2}(n_ik_{i+1}-n_{i+1}k_i)\right]\right\}.\end{aligned}$$

We can now perform the sum over $\vec{k}$ in the above equation by using the definition of the Jacobi theta function

$$\vartheta(\nu,\tau) = \sum_{n=-\infty}^{\infty}\exp(\pi i n^2\tau + 2i\pi n\nu)\,. \tag{2.6}$$

Then using the property of modular transformation [2, 14]

$$\vartheta(\nu,\tau) = (-i\tau)^{-1/2}\exp\left(\frac{-\pi i\nu^2}{\tau}\right)\vartheta\left(\frac{\nu}{\tau},\frac{-1}{\tau}\right). \tag{2.7}$$

we can express the result as

$$\begin{aligned}\Sigma(p,\vec{n}) &= -\frac{\lambda^2}{4}\frac{1}{(2\pi R)^{2L}}\frac{1}{\beta}\frac{1}{(4\pi)^{2/d}}\sum_{l_0}\int_0^1 dx\int_0^\infty d\alpha\alpha^{1-d/2}\left(\frac{\alpha}{\pi R^2}\right)^{-L}\left(\frac{\beta^2}{4\pi\alpha}\right)^{1/2}\times\\ &\times\exp\left\{-\alpha\left\{m^2+x(1-x)\left[\left(\frac{\beta p_0}{2\pi}\right)^2+\vec{p}^2+\frac{\vec{n}^2}{R^2}\right]\right\}-\frac{1}{\alpha}\frac{\theta^2\vec{n}^2}{4R^2}\right\}\times\end{aligned}$$

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$$\times \exp\left\{-\frac{\beta^2}{4\alpha}l_0^2 + i2\pi x p_0 l_0\right\} \prod_{j=1}^{2L-1} \vartheta\left(x n_j + i\frac{n_{j+1}}{2\alpha}, i\frac{\pi R^2}{\alpha}\right) \times$$
$$\times \vartheta\left(x n_{j+1} - i\frac{n_j}{2\alpha}, i\frac{\pi R^2}{\alpha}\right).$$

From the above equation we see that the ultraviolet divergent contribution comes from $\alpha \to 0$ region. Thus the leading contribution will come from the region $\alpha \to 0$ [15] and we can set $\vartheta = 1$ in the above equation. The leading correction to the spectrum of Kaluza-Klein system becomes

$$\Sigma(p,\vec{n}) \approx -\frac{\lambda^2}{4}\frac{1}{(4\pi)^{L+\frac{d+1}{2}}}\sum_{l_0}\int_0^1 dx \int_0^\infty d\alpha \alpha^{1-L-\frac{1+d}{2}} \exp\left\{-\frac{1}{\alpha}\frac{\theta^2\vec{n}^2}{4R^2}\right\} \times$$
$$\times \exp\left\{-\frac{\beta^2}{4\alpha}l_0^2 + i2\pi x p_0 l_0\right\}$$
$$= -\frac{\lambda^2}{4}\frac{1}{(4\pi)^{L+\frac{d+1}{2}}}\sum_{l_0}\int_0^1 dx \frac{\Gamma(L+\frac{d-3}{2})}{[\frac{\beta^2 l_0^2}{4} + \frac{\theta^2\vec{n}^2}{4R^2}]^{L+\frac{d-3}{2}}} \exp(i2\pi x p_0 l_0). \qquad (2.8)$$

This is the main result of this section.

2.3 Self energy

Let us first examine the above result at zero temperature. In this case $\beta \to \infty$ and only the mode $l_0 = 0$ contribute the summation in eq. (2.8). Thus we can easily find that

$$\Sigma(p,\vec{n}) = -\frac{\lambda^2}{64}\frac{\Gamma(L+\frac{d-3}{2})}{\pi^{L+\frac{d+1}{2}}}\left(\frac{R^2}{\theta^2\vec{n}^2}\right)^{L+\frac{d-3}{2}}, \qquad \text{if } T = 0, \qquad (2.9)$$

which is that derived by us in [16] and reduces to that in [15] when $L = 1$ and $d = 3$.

From eq. (2.8) we also see that if $p_0 \neq 0$ (note that both of p_0 and l_0 are integral) then the integration over x will becomes zero unless $l_0 = 0$. Thus we have another relation

$$\Sigma(p,\vec{n}) = -\lambda_p^2\left(\frac{R^2}{\theta^2\vec{n}^2}\right)^{L+\frac{d-3}{2}}, \qquad \lambda_p^2 \equiv \frac{\lambda^2}{64}\frac{\Gamma(L+\frac{d-3}{2})}{\pi^{L+\frac{d+1}{2}}}, \qquad \text{if } p_0 \neq 0. \qquad (2.10)$$

When $p_0 = 0$ then, as $\beta \to 0$ in the high-temperature limit, the summation over l_0 in eq. (2.8) could be replaced by an integration. The result is

$$\Sigma(p,\vec{n}) = -\frac{\lambda_0^2}{\beta}\left(\frac{R^2}{\theta^2\vec{n}^2}\right)^{L+d/2-2}, \qquad \lambda_0^2 \equiv \frac{\lambda^2}{64}\frac{\Gamma(L+d/2-2)}{\pi^{L+d/2}},$$
$$\text{if } T \gg 1, \quad p_0 = 0. \qquad (2.11)$$

The relation (2.9) have been used to evaluated the zero-temperature Casimir energy in [16]. We will in next section use the relations (2.10) and (2.11) to evaluated the Casimir energy in the high-temperature limit.

Note that from eqs. (2.10) and (2.11) we see that when $L + \frac{d-3}{2} = 1$ in the case of $p_0 \neq 0$ or $L + d/2 - 2 = 1$ in the case of $p_0 = 0$, then the winding states like as those in the string theory will appear [20]. The property had been found in the zero temperature system [15].

3. Casimir energy at finite temperature

3.1 Partition function, zeta function and Casimir energy

For the model (2.1) in thermal equilibrium at a finite temperature the partition Z for the system can be evaluated from the relation [21]

$$\ln Z = -\frac{1}{2}\zeta_H'(0)\,. \tag{3.1}$$

in which the zeta function is defined by

$$\zeta_H(\nu) = \frac{1}{\Gamma(\nu)}\int ds\, s^{\nu-1}\sum_{p_0}\sum_{\vec{n}}\int d^d\vec{p} e^{-sH}\,, \tag{3.2}$$

where $H = H_0 + \Sigma(p,\vec{n})$, $H_0 = (2\pi p_0/\beta)^2 + \vec{p}^2 + \vec{n}^2/R^2$ and $\Sigma(p,\vec{n})$ is defined by (2.10) and (2.11). The zeta function can be evaluated with the help of the Schwinger perturbative formula [20]

$$\zeta_H(\nu) = \zeta_0(\nu) + \zeta_1(\nu)\,, \tag{3.3}$$

in which

$$\zeta_0(\nu) = \frac{1}{\Gamma(\nu)}\int ds\, s^{\nu-1}\sum_{p_0}\sum_{\vec{n}}\int d^d\vec{p} e^{-sH_0}\,, \tag{3.4}$$

$$\zeta_1(\nu) = \frac{1}{\Gamma(\nu)}\int ds\, s^{\nu-1}\sum_{p_0}\sum_{\vec{n}}\int d^d\vec{p}(-s\Sigma(p,\vec{n}))e^{-sH_0}\,. \tag{3.5}$$

Using the usual formula [1, 2]

$$E = \frac{\partial}{\partial\beta}\ln Z\,. \tag{3.6}$$

the Casimir energy is obtained.

3.2 Calculation of zeta function

Let us first evaluate the $\zeta_0(\nu)$. We can first perform the integration over the momentum $\vec{p}$ and then the variable s. The result is

$$\begin{aligned}
\zeta_0(\nu) &= \frac{1}{\Gamma(\nu)}\int ds\, s^{\nu-1}\sum_{p_0}\sum_{\vec{n}}\int d^d\vec{p}\,\exp\left[-s\left(\left(\frac{2\pi p_0}{\beta}\right)^2 + \vec{p}^2 + \frac{\vec{n}^2}{R^2}\right)\right]\\
&= \pi^{d/2}\frac{\Gamma(\nu - d/2)}{\Gamma(\nu)}\sum_{p_0}\sum_{\vec{n}}\left[\left(\frac{2\pi p_0}{\beta}\right)^2 + \frac{\vec{n}^2}{R^2}\right]^{d/2-\nu}\\
&\approx \nu\left[2^{2L+d+1}\pi^{-1/2}\beta^{-2L-d}\Gamma\left(\frac{2L+d+1}{2}\right)\zeta(2L+d+1)\right] + O(\nu^2)\,,
\end{aligned} \tag{3.7}$$

where $\zeta(N)$ is the Riemann zeta function which is only divergent at $N = 1$. Note that to obtain the final relation we have used the formula A(12) in ref. [2] to take the summations over p_0 and $\vec{n}$ in the high-temperature limit. Substituting the above result into eqs. (3.1) and (3.6) we see that the zero-order Casimir force (i.e. without noncommutativity) is attractive for arbitrary values of L and d.

To calculate $\zeta_1(\nu)$ we first consider that contributes from $p_0 = 0$. From eqs. (2.11) and (3.5) we have

$$\zeta_1^{p_0=0}(\nu) = \frac{1}{\Gamma(\nu)} \int_0^\infty ds\, s^\nu \sum_{\vec{n}} \int d^d\vec{p}\, \frac{\lambda_0^2}{\beta} \left(\frac{R^2}{\theta^2 \vec{n}^2}\right)^{L+\frac{d}{2}-2} \exp\left[-s\left(\vec{p}^2 + \frac{\vec{n}^2}{R^2}\right)\right],$$
$$= \nu \left[\frac{1}{\beta} \frac{\lambda_0^2}{(\theta^2)^{L-2+d/2}}\, \pi^{d/2}\Gamma\left(1+\frac{d}{2}\right) R^{2L-2} \sum_{\vec{n}} \left(\frac{1}{\vec{n}^2}\right)^{L-1}\right] + O(\nu^2)\,. \qquad (3.8)$$

Substituting the above result into eqs. (3.1) and (3.6) we see that the contribution from $p_0 = 0$ in the first-order Casimir force is attractive for arbitrary value d if $L > 1$. When $L = 1$ the Casimir energy is R independent and there is no Casimir force.

Let us turn to the contribution from $p_0 \neq 0$. We first from the eqs. (2.10) and (3.5) perform the integration over $\vec{p}$. Then after introducing a variable x we can perform the integration over the variable s and then the variable x. The result is

$$\zeta_1^{p_0\neq 0}(\nu) = \frac{\lambda_\theta^2}{\Gamma(\nu)} \int_0^\infty ds\, s^\nu \sum_{\vec{n}} \sum_{p_0\neq 0} \int d^d\vec{p} \left(\frac{R^2}{\theta^2\vec{n}^2}\right)^{L+\frac{d-3}{2}} \times$$
$$\times \exp\left[-s\left(\left(\frac{2\pi p_0}{\beta}\right)^2 + \vec{p}^2 + \frac{\vec{n}^2}{R^2}\right)\right]$$
$$= \frac{\lambda_\theta^2 \pi^{d/2}}{\Gamma(\nu)\Gamma(L+\frac{d-3}{2})} \int_0^\infty dx x^{L+\frac{d-5}{2}} \int_0^\infty ds\, s^{L+\nu-3/2} \times$$
$$\times \sum_{\vec{n}} \sum_{p_0\neq 0} \frac{\lambda_\theta^2}{\beta} \left(\frac{R^2}{\theta^2\vec{n}^2}\right)^{L+\frac{d-3}{2}} \exp\left[-s\left(\left(\frac{2\pi p_0}{\beta}\right)^2 + x\frac{\vec{n}^2}{R^2}\right)\right]$$
$$= \nu\left[\lambda_\theta^2\, \frac{1}{\Gamma(L-d-1/2)}\, \beta^{2-3d}\, R^{2L-2}\right] + O(\nu^2), \qquad (3.9)$$

where

$$\lambda_\theta^2 \equiv \frac{\lambda^2}{(\theta^2)^{L+\frac{d-3}{2}}}\, \pi^{L+5/2}\, 2^{3d-6}\, \Gamma\left(\frac{3d-1}{2}\right) \Gamma\left(L+\frac{d-3}{2}\right) \sum_{\vec{n}} \left(\frac{1}{\vec{n}^2}\right)^{2L+d-3}, \qquad (3.10)$$

is positive. Note that to obtain the above result we have use the reflation formula [2]

$$\pi^{-z/2}\,\Gamma\left(\frac{z}{2}\right)\,\zeta(z) = \pi^{-\frac{1-z}{2}}\,\Gamma\left(\frac{1-z}{2}\right)\,\zeta(1-z)\,, \qquad (3.11)$$

to regularize the divergence in the original relation.

Substituting the above result into eqs. (3.1) and (3.6) we see that the contribution from $p_0 \neq 0$ in the first-order Casimir force may be attractive or repulsive, which depend on the values of L and d. This is because that the Gamma function $\Gamma(L-d-1/2)$ in (3.9) can become negative if $d-L$ is a non-negative even integral. Not also that, as that in the case of $p_0 = 0$, when $L = 1$ the Casimir energy is R independent and there is no Casimir force.

3.3 Compactification radius

Therefore, when $L > 1$ and $d-L$ is a non-negative even integral, we can substituting eqs. (3.7), (3.8) and (3.9) into eqs. (3.1) and (3.6) to find the Casimir energy. From the Casimir energy we can find the Casimir force and find the compactification radius R_T. In the high-temperature limit we have the final result

$$R_T = \left[- \frac{(3d-2)(L-1)\pi^{1/2}\,\lambda_\theta^2\,\beta^{2L+2-2d}}{(2L+d)\,2^{2L+d+2}\,\Gamma(\frac{2L+d+1}{2})\,\Gamma(L-d-1/2)\zeta(L+d+1)} \right]^{1/(2L+2)} . \quad (3.12)$$

Thus we see that the Casimir energy could give repulsive force to stabilize the extra non-commutative torus. This therefore suggests a possible stabilization mechanism for a high-temperature scenario in the Kaluza-Klein theory, where some of the extra dimensions are non commutative.

4. Conclusion

he appearance of the parameter of noncommutativity in the context of string theory [8] seems to be a good opportunity to provide a minimum scale to protect the collapse of the extra spaces. This is because that the commutation relation $[x^\mu, x^\nu] = i\theta^{\mu\nu}$ leads to the spacetime-time uncertain relation $\delta x^\mu \delta x^\nu \geq \frac{1}{2}\theta^{\mu\nu}$. However, the noncommutativity will in general break the unitarity [21] and Lorentz invariant [8]. Therefore, in this paper we choose the spacetime in which noncommutativity only shows in the extra space. The result theory is unitary with the observable spaces which are Lorentz invariant, as that consider by Gomis, Mehen and Wise in ref. [15], and could be relevant phenomenologically

The investigation of Casimir effect in the non-zero temperature is physically interesting, as the temperature can break the supersymmetry and Casimir energy contribution of boson field will not be canceled by that of the fermion field. The temperature in the early epoch of the universe is very high and may relate to the present paper.

Our evaluations have shown that the Casimir force is attract if the extra spaces are commutative. However, when the L extra two-dimensional tori become non commutative, then it could contribute a repulsive force to protect the collapse of the extra spaces. We have found that the one-loop Casimir energy is independent of the radius of torus if $L = 1$. So, in this case, there is no repulsive force and we have to consider the Casimir energy to the higher order, which is beyond the scope of this paper. However, when $L > 1$ the Casimir energy could give repulsive force to stabilize the extra non-commutative torus if $d - L$ is a non-negative even integral. We therefore have found a possible stabilization mechanism of extra radius in high temperature, when the extra spaces are non commutative.

Finally, we want to mention an interesting phenomena about our investigations. At zero temperature, the previous work [16] found that when $L > 1$ then the Casimir energy due to the noncommutativity could give repulsive force to stabilize the extra non-commutative tori in the cases of $d = 4n-2$, with n a positive integral. Comparing this with the present work at finite temperatureone, we see that the non-commutative tori which will collapse at zero temperature may, due to finite-temperature Casimir effect, have a finite stable radius at high temperature. On the other hand, that having a stable radius at zero temperature may collapse at high temperature. The behavior depends on the values of L and d. Such a dramatic change, however, dose not show in the commutative space. Therefore, the spaces which are non-commutative may have phase transition at a finite temperature. The transition is due to the finite-temperature Casimir effect on a non-commutative geometry.

References

[1] H.B.G. Casimir, *On the attraction between two perfectly conducting plates*, *Proc. K. Ned. Akad. Wet.* **51** (1948) 793;
G. Plunien, B. Muller and W. Greiner, *The Casimir effect*, *Phys. Rep.* **134** (1986) 87;
V.M. Mostepanenko and N.N. Trunov, *The Casimir effect and its applications*, Oxford University Press, 1997.

[2] J. Ambjørn and S. Wolfram, *Properties of the vacuum, 1. Mechanical and thermodynamic*, *Ann. Phys. (NY)* **147** (1983) 1.

[3] M.J. Sparnaay, *Measurements of attractive forces between flat plates*, *Physica* **24** (1958) 751;
S.K. Lamoreaux, *Demonstration of the Casimir force in the 0.6 to 6 micrometers range*, *Phys. Rev. Lett.* **78** (1997) 5.

[4] T. Appelquist and A. Chodos, *Quantum effects in Kaluza-Klein theories*, *Phys. Rev. Lett.* **50** (1983) 141; *The quantum dynamics of Kaluza-Klein theories*, *Phys. Rev.* **D 28** (1983) 772.

[5] Th. Kaluza, *On the unification problem in physics*, *Situngsber. d. K. Preuss. Akad. d. Wissen. z. Berlin*, Phys.-Math. Klasse (1921) 966; O. Klein, *Z. F. Physik* **37** (1926) 895;
T. Appelquist, A. Chodos, and P.G.O. Freund, *Modern Kaluza-Klein theories*, Addison-Wesley, Menlo Park, 1987.

JHEP11(2000)041

[6] M.B. Green, J.H. Schwarz and E. Witten, *Superstring theory*, Cambridge University Press, Cambridge 1987.

[7] A. Connes, M.R. Douglas and A. Schwarz, *Noncommutative geometry and matrix theory: compactification on tori*, *J. High Energy Phys.* **02** (1998) 003 [hep-th/9711162];
B. Morariu and B. Zumino, *Super Yang-Mills on the noncommutative torus*, hep-th/9807198;
C. Hofman and E. Verlinde, *U-duality of Born-Infeld on the noncommutative two-torus*, *J. High Energy Phys.* **12** (1998) 010 [hep-th/9810116].

[8] F. Ardalan, H. Arfaei and M.M. Sheikh-Jabbari, *Noncommutative geometry from strings and branes*, *J. High Energy Phys.* **02** (1999) 016 [hep-th/9810072]; *Dirac quantization of open strings and noncommutativity in branes*, *Nucl. Phys.* **B 576** (2000) 578 [hep-th/9906161];
M.M. Sheikh-Jabbari and A. Shirzad, *Boundary conditions as Dirac constraints*, hep-th/9907055;
N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *J. High Energy Phys.* **09** (1999) 032 [hep-th/9812219];
C.C. Chu and P.M. Ho, *Noncommutative open string and D-brane*, *Nucl. Phys.* **B 550** (1999) 151 [hep-th/9906192],
T. Lee, *Canonical quantization of open string and noncommutative geometry*, *Phys. Rev.* **D 62** (2000) 024022 [hep-th/9911140].

[9] N.A. Obers and B. Pioline, *U-duality and M-theory*, *Phys. Rep.* **318** (1999) 113 [hep-th/9809039].

[10] J. Polchinski, *String theory*, Cambridge University Press, 1998.

[11] H.S. Snyder, *Quantized space-time*, *Phys. Rev.* **71** (1947) 38;
Phys. Rev. **72** (1947) 68;
A. Connes, *Noncommutative geometry* Academic Press, New York, 1994;
G. Landi, *An introduction to noncommutative spaces and their geometries*, Lecture Notes in Physics, Springer-Verlag, [hep-th/9701078];
J. Várilly, *An introduction to noncommutative geometry*, Summer School *Noncommutative geometry and applications*, Lisbon, September 1997, physics/9709045.

[12] T. Filk, *Divergencies in a field theory on quantum space*, *Phys. Lett.* **B 376** (1996) 53;
S. Minwalla, M.V. Raamsdonk and N. Seiberg, *Noncommutative perturbative dynamics*, hep-th/9912072;
C.P. Martin and D. Sanchez-Ruiz, *The one-loop UV divergent structure of* U(1) *Yang-Mills theory on noncommutative* R^4, *Phys. Rev. Lett.* **83** (1999) 476 [hep-th/9903077];
I. Chepelev and R. Roiban, *Renormalization of quantum field theories on noncommutative* R^d*, 1. Scalars*, *J. High Energy Phys.* **05** (2000) 037 [hep-th/9911098];
B.A. Campbell and K. Kaminsky, *Noncommutative field theory and spontaneous symmetry breaking*, *Nucl. Phys.* **B 581** (2000) 240 [hep-th/0003137];

A. Matusis, L. Susskind and N. Toumbas, *The IR/UV connection in the non-commutative gauge theories*, hep-th/0002075.

[13] W.-H. Huang, *Two-loop effective potential in noncommutative scalar field theory*, *Phys. Lett.* **B 496** (2000) 206 [hep-th/0009067].

[14] S. Nam, *Casimir force in compact noncommutative extra dimensions and radius stabilization*, *J. High Energy Phys.* **10** (2000) 044 [hep-th/0008083].

[15] J. Gomis, T. Mehen and M.B. Wise, *Quantum field theories with compact noncommutative extra dimensions*, *J. High Energy Phys.* **08** (2000) 029 [hep-th/0006160].

[16] W.-H. Huang, *Casimir effect on the radius stabilization of the noncommutative torus*, hep-th/0010160.

[17] S.J. Gates, H.T. Grisaru, M. Rocek and W. Siegel, *Superspace: one thousand and one lectures in supersymmetry*, Benjamin/Cummings, 1983;
P. West, *Introduction to the supersymmetry and supergavity*, World Scientific Publishing, 1986.

[18] B.A. Ovrut and J. Wess, *Supersymmetric R(XI) gauge and radiative symmetry breaking*, *Phys. Rev.* **D 25** (1982) 409;
A. Sen, *Radiative corrections in supersymmetric gauge theories*, *Phys. Rev.* **D 31** (1985) 2100;
W.-H. Huang, *On the no go theorem of supersymmetry breaking*, *Phys. Lett.* **B 179** (1986) 92.

[19] A. Das and M. Kaku, *Supersymmetry at high temperatures*, *Phys. Rev.* **D 18** (1978) 4540;
L. Girardello, M.T. Grisaru and P. Salomonson, *Temperature and supersymmetry*, *Nucl. Phys.* **B 178** (1981) 331.

[20] W. Fischler, E. Gorbatov, A. Kashani-Poor, S. Paban, P. Pouliot and J. Gomis, *Evidence for winding states in noncommutative quantum field theory*, *J. High Energy Phys.* **05** (2000) 024 [hep-th/0002067].

[21] J. Schwinger, *On gauge invariance and vacuum polarization*, *Phys. Rev.* **82** (1951) 664;
L. Parker, in *Recent development in gravitation*, Deser and M. Levey eds., Plenum Press, New York 1977;
W.-H. Huang, *One loop effective action on rotational space-time: zeta-function regularization and Schwinger perturbative expansion*, *Ann. Phys. (NY)* **254** (1997) 69; *Semiclassical gravitation and quantization for the Bianchi type-I universe with large anisotropy*, *Phys. Rev.* **D 58** (1998) 084007.

[22] J. Gomis and T. Mehen, *Space-time noncommutative field theories and unitarity*, *Nucl. Phys.* **B 591** (2000) 265 [hep-th/0005129].

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